

**Proportional and Scale Change Models to Project Failures of
Mechanical Components with Applications to Space Station**

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ABSTRACT

In this paper we develop the mathematical theory of proportional and scale change models to perform reliability analysis. The results obtained will be applied for the reaction Control System (RCS) thruster valves on an orbiter. With the advent of extended EVA's associated with PROX OPS (ISSA & MIR), and docking, the loss of a thruster valve now takes on an expanded safety significance. Previous studies assume a homogeneous population of components with each component having the same failure rate. However, as various components experience different stresses and are exposed to different environments, their failure rates change with time.

In this paper we model the reliability of a thruster valves by treating these valves as a censored repairable system. The model for each valve will take the form of a nonhomogeneous process with the intensity function that is either treated as a proportional hazard model, or a scale change random effects hazard model. Each component has an associated z , an independent realization of the random variable Z from a distribution $G(z)$. This unobserved quantity z can be used to describe heterogeneity systematically.

For various models methods for estimating the model parameters using censored data will be developed. Available field data (from previously flown flights) is from non-renewable systems. The estimated failure rate using such data will need to be modified for renewable systems such as thruster valve.

INTRODUCTION

In this paper we develop the mathematical theory of proportional and scale change models to perform reliability analysis. The results obtained will be applied for the Reaction Control System (RCS) thruster valves on a space vehicle. With the advent of extended EVA's associated with PROX OPS (ISSA & MIR), and docking to space station, the loss of a thruster valve now takes on an expanded safety significance. RCS thruster valves are installed on the orbiter at 38 locations, 14 in front (Forward) and 24 in the rear (Aft), 12 on each side. At each location there is a fuel valve and an oxidizer valve. Thus there are a total of 76 valves on an orbiter. These inlet valves can leak due to various reasons. These include the shrinkage of the teflon seal due to extreme weather conditions, reduction in the teflon seal height above the seal retainer, and contamination deposits between the valve seat and poppet face. The mixing of moist air and residual oxidizer (N_2O_4) form metallic nitrates. It is believed that the metallic nitrates cause deposits to build up in the valves.

The orbiter thruster valves have at least three failures modes. These are: (1) Nitrate build up so that the valve will not open (FAIL-OFF/CLOSED); (2) Nitrate deposits on the seat causes leaks and the valve will not close (FAIL-OFF/OPEN); and (3) Spontaneous leaks (FAIL OFF/LEAKS). The number of times a valve is opened or closed provide an indication of the amount of fluid flow which may be related to the contamination failure mechanism. Also, the amount of fluid each valve is subjected to varies substantially from each location. In the past several studies have been done in an attempt to estimate valve reliability. Studies done at Rockwell Aerospace have used cycle time as the casual variable, while studies done at JSC have used soak time as the casual variable. Only one variable was used in both of these studies since standard statistical computer models treat only one variable. In this paper we develop new statistical theory based on both variables. Also, previous studies assume a homogeneous population of components with each component having the same failure rate. However, as various components experience different stresses and are exposed to different environments, their failure rates can change across the population of components. Techniques which ignore the heterogeneity can result in incorrect estimates of failure distributions.

We propose to model the reliability of the thruster valves by treating these valves as a censored repairable system. The system is repairable since valves that either leak or stick are removed, repaired and placed back in operation. Censoring occurs whenever the time-to-failure records are terminated before each valve has had a chance to fail. The model for each valve will take the form of a nonhomogeneous process with the intensity function that is either treated as a proportional hazard model, or a scale change random effects hazard model.

The proportional hazard function model will assume that the time the valve is soaked in the oxidizer prior to failure is the primary variable that describes the base hazard function and that the cycle time, and perhaps other variables, adjust this hazard. The scale change random effects model will also assume that there is a soak-time-hazard function for each valve, but in this case cycle time and other variables will be used to randomly scale the soak time.

Thus for a given $Z=z$, the cumulative hazard function for proportional change and scale change models are given respectively by,

$$H(t / Z=z) = z H(t), \quad \text{and} \quad H(t / Z=z) = H(zt),$$

where $H(t)$ is an unobserved cumulative baseline hazard function. Thus each component has an associated z , an independent realization of the random variable Z from a distribution $G(z)$. This unobserved quantity z can be used to describe heterogeneity systematically. This variable z may represent environmental influences on different components, effects of microgravity, effect of location of components on the orbiter, and various other risk factors.

For each of these models methods for estimating the model parameter using censored field data will be developed. The model which appears to best forecast failures of the Orbiter's RCS thruster valves will then be selected as the appropriate model.

To estimate the component life for components on a space vehicle (such as ISSA), one needs to understand the mechanism that cause the failures of the components and component types. Ideally each component with a different vintage should be put on test under environmental and operational conditions identical to those under which it is to be operated, and time to failure be observed. This experiment needs to be repeated a number of times to get a reasonable size statistical sample. However it is not possible to conduct meaningful life tests on earth because of not being able to replicate the proper stress environment and also because of cost. Thus, the only available data on failure of components in microgravity is the field data obtained from previously flown spacecraft's. This data needs to be adjusted because:

1. The available data is from non-renewable systems, i.e., a failed component is not replaced. The failure rate distributions estimated using such data will need to be modified for renewable systems such as ISSA.
2. Previous studies use the field data collected from sixties, seventies and eighties, and conclude that design and environment are the main contributors to failure. Assuming that how to design and knowledge about environment has improved substantially since sixties, this data need to be examined carefully.

MIXTURE MODELS UNDER HETEROGENEITY

It is generally accepted that the lifetime of electronic components can be described by an exponential probability distribution, that is,

$$f(t) = \lambda \cdot e^{-\lambda t}, t \geq 0, \lambda > 0 \quad (1)$$

where λ is a parameter which is the hazard rate. This model assumes constant failure rate for each component. However, in most cases the failure rate-age characteristic may rise or fall in addition to remaining constant. A probability distribution which can represent any form of failure rate-age curve is the Weibull distribution. The probability density function of this distribution is given by

$$f(t) = \frac{\beta}{\eta} \left(\frac{t}{\eta}\right)^{\beta-1} \cdot e^{-\left(\frac{t}{\eta}\right)^\beta}, \quad t \geq 0, \beta, \eta > 0 \quad (2)$$

Exponential distribution (1) is a special case when $\beta = 1$. The cumulative distribution function, survival function $S(t)$, and hazard function are given respectively by

$$F(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^\beta}; S(t) = 1 - F(t); \text{ and } h(t) = \frac{\beta}{\eta} \left(\frac{t}{\eta}\right)^{\beta-1}$$

These models assume a homogeneous population of components with each component having the same failure rate. However, as various components on the orbiter are exposed to different environments and experience different stresses, their failure rates can change across the population of component types. In this paper we develop proportional and scale change models when the life-time distribution is given by (2). The failure model for each component will take the form of a non-homogeneous process with intensity function that is either treated as a proportional hazard model, or a scale change random effects hazard model.

Proportional Hazard Model

Under this model the lifetime of a component, T , has the cumulative hazard function, $H(t / Z=z) = z H(t)$, where $H(t)$ is an unobserved baseline cumulative hazard function, the same for each component. Each component has an associated z , an independent realization of a r.v. Z from a distribution $G(z)$. This variable z (possibly a vector) can represent environmental influences on different components, effects of microgravity, effects of location on the orbiter, and various other risks factors. This unobserved quantity z can be used to describe heterogeneity systematically.

Case I:

Let $g(z) = \frac{\lambda}{\Gamma(r)} (\lambda \cdot z)^{r-1} e^{-\lambda z}$, $z \geq 0$, $\lambda, \dots, r > 0$, then, (3)

$$S(t) = E_z S(t/Z=z) = \int_0^{\infty} e^{-\lambda(\frac{t}{\eta})^\beta} \frac{\lambda}{\Gamma(r)} (\lambda \cdot z)^{r-1} e^{-\lambda z} dz$$

$$= \frac{\lambda^r}{\left[\lambda + \left(\frac{t}{\eta}\right)^\beta\right]^r},$$

and,

$$h(t) = -\frac{d}{dt} \log S(t) = \frac{r}{\eta} \frac{\beta \left(\frac{t}{\eta}\right)^{\beta-1}}{\lambda + \left(\frac{t}{\eta}\right)^\beta} \quad (4)$$

when $t = \eta$, the characteristic life is given by,

$$h(t) = \frac{r}{\eta} \frac{\beta}{\eta+1}$$

We note from (4) that for given r, β, η, λ , $h(t) \mapsto 0$ as t gets large.

Case II:

Let $g(z) = 1$, $0 < z < 1$. (5)

In this case,

$$S(t) = \frac{1 - e^{-\left(\frac{t}{\eta}\right)^\beta}}{\left(\frac{t}{\eta}\right)^\beta}; \text{ and } h(t) = \frac{\beta}{t} - \frac{\beta \cdot \left(\frac{t}{\eta}\right)^{\beta-1} \cdot e^{-\left(\frac{t}{\eta}\right)^\beta}}{1 - e^{-\left(\frac{t}{\eta}\right)^\beta}}. \quad (6)$$

We note that $h(t) \mapsto 0$ as t gets large.

Case III: Let $g(z)$ be a 2-point distribution, i.e.

$$P(Z=z_1) = p; P(Z=z_2) = 1-p \quad (7)$$

then,

$$S(t) = p e^{-z_1 \left(\frac{t}{\eta}\right)^\beta} + (1-p) \cdot e^{-z_2 \left(\frac{t}{\eta}\right)^\beta}.$$

The expression for the hazard rate, $h(t)$, is lengthy, but can be obtained easily. Also, it can be shown that $h(t) \mapsto 0$, as t gets large.

Scale change Hazard Models

Under the scale change hazard model the lifetime, T , has the cumulative hazard function, $H(t / Z=z) = H(zt)$, where $H(t)$ is an unobserved baseline hazard function, the same for all components. In this case,

$$S(t / Z=z) = e^{-H(zt)} \quad \text{and} \quad S(t) = \int_0^{\infty} e^{-H(zt)} dG(z), \quad (8)$$

Case I:

Let Z has the gamma distribution given by (3), then from (8)

$$S(t) = \int_0^{\infty} e^{-z^{\beta}(\frac{t}{\eta})^{\beta}} \cdot \frac{\lambda}{\Gamma(r)} (\lambda \cdot z)^{r-1} e^{-\lambda z} dz, \quad \text{and} \quad h(t) = -\frac{d}{dt} \log S(t) \quad \text{can be computed and}$$

the resulting integral can be evaluated numerically.

Case II:

Let Z has the Weibull distribution given by (2) with η replaced by δ . In this case,

$$S(t) = \frac{1}{\delta^{\beta} \left[\left(\frac{t}{\eta}\right)^{\beta} + \left(\frac{t}{\delta}\right)^{\beta} \right]}, \quad \text{and} \quad h(t) = \frac{\frac{\beta}{\eta} \left(\frac{t}{\eta}\right)^{\beta-1}}{\left(\frac{t}{\eta}\right)^{\beta} + \left(\frac{t}{\delta}\right)^{\beta}} \quad (9)$$

we note that $h(t) \mapsto 0$ as t gets large.

Case III:

Let Z has the uniform distribution given by (5). In this case,

$$S(t) = \int_0^1 e^{-z^{\beta}(\frac{t}{\eta})^{\beta}} dz, \quad \text{and} \quad h(t) = \frac{1}{t} \left[1 - \frac{e^{-\left(\frac{t}{\eta}\right)^{\beta}}}{S(t)} \right]. \quad (10)$$

$S(t)$ needs to be evaluated numerically. It can be shown that $h(t) \mapsto 0$ as t gets large.

Case IV:

Let Z has the 2-point distribution given by (7). In this case

$$h(t) = \beta \cdot t^{\beta-1} \left[p z_1^{\beta} e^{-z_1^{\beta}(\frac{t}{\eta})^{\beta}} + (1-p) z_2^{\beta} e^{-z_2^{\beta}(\frac{t}{\eta})^{\beta}} \right], \quad (11)$$

Where z_1, z_2 are particular values of Z . It is clear from (11) that $h(t) \rightarrow 0$ as t gets large.

A Regression Model

Assume that for a given value z of a random variable Z , the regression of $\log T$ on $\log z$ is linear, that is,

$$\log T = \log a + b \log z + \log \varepsilon \quad (12)$$

where a is a constant. From (12) $T = a z^b \varepsilon$, $S(t/z) = P(T > t/z) = P(\varepsilon > \frac{t}{a z^b} / z)$, and

$$S(t) = \int P(\varepsilon > \frac{t}{a z^b}) \cdot g(z) dz, \quad (13)$$

where $g(z)$ is the density of Z . This integral can be evaluated either analytically or numerically depending on the form of the densities of ε and z . Assuming ε has a Weibull (β, η) distribution, the survival function $S(t)$ is given by,

$$S(t) = \int e^{-\left(\frac{t}{a z^b}\right)^\beta} g(z) dz = \int e^{-\Psi(z) \cdot H(t)} g(z) dz,$$

where $g(z)$ is the density of z , $\Psi(z)$ is a function of z only, and $H(t)$ does not depend on z . We note that both proportional hazard and scale change hazard models are special cases, when $\Psi(z) = z$, and $\Psi(z) = z^\beta$ respectively.

Case 1:

Let $g(z) = \frac{\phi^\lambda}{\Gamma(\lambda)} \cdot \left(\frac{1}{z}\right)^{\lambda+1} e^{-\frac{\phi}{z}}$ (inverted gamma). In this case survival function $S(t)$ is given by

$$S(t) = \frac{\phi^\lambda}{\Gamma(\lambda)} \int_0^\infty e^{-\left[\left(\frac{t}{a z^b}\right)^\beta + \frac{\phi}{z}\right]} \left(\frac{1}{z}\right)^{\lambda+1} dz \quad (14)$$

This integral can be evaluated numerically. In a special case when $b\beta=1$, the expression in (14) can be simplified, that is,

$$S(t) = \left(\frac{\phi \cdot \mu}{t^{\frac{1}{b}} + \phi \cdot \mu} \right)^\lambda, \quad \text{where } \mu = (a\eta)^{\frac{1}{\beta}}. \text{ From this, we get}$$

$$h(t) = \frac{\lambda \cdot t^{\frac{1}{b}-1}}{b(t^{\frac{1}{b}} + \phi \cdot \mu)}, \quad (15)$$

which approaches zero as t gets large

Case II:

If $\log \varepsilon$ is distributed as a $n(0, \sigma^2)$, then ε has the lognormal distribution. In this case,

$$S(t/z) = P(T > t/z) = P(\varepsilon > \frac{t}{a \cdot z})$$

$$= 1 - \Phi\left[\frac{1}{\sigma}(\log t - \log a - b \log z)\right],$$

where $\Phi(z)$ is the distribution function of the standard normal, and we get

$$S(t) = \int_0^{\infty} \int_0^{\infty} \frac{1}{\sqrt{2\pi} \cdot \sigma(x + \log \frac{t}{a \cdot z})} e^{-\frac{1}{2\sigma^2}(\log(x + \log \frac{t}{a \cdot z}))^2} g(z) dx dz, \quad (16)$$

where $x = \varepsilon - \log \frac{t}{a \cdot z}$. This integral can be computed numerically, and then $h(t)$ can be computed.

Applications to RCS Thruster Valves

As stated in the introduction, previous studies to estimate the reliability of a RCS thruster valve use either cycle time or soak time as the casual variable. It is quite possible that each of these two variables has a substantial effect on the valve contamination mechanism. It is also possible that other factors such as, location of valve on the orbiter, also contribute to failure mechanism. A preliminary analysis using logistic function can be used to decide which of the two independent variables has major effect on the contamination failure mechanism. Let S denote the Soak time and C denote the cycle time, and define $P(s,c)$ denote the probability of a valve failure for a given $S=s$ and $C=c$.

The logistic regression model assumes that the $\log(\text{odds})$ is a linear function of independent variables. This procedure can be performed as follows:

Step 1.

Use data to fit 3 logistic functions independently:

$$\log(p(s,c) / (1-p(s,c))) = \alpha + \beta s + \phi c \quad (17)$$

$$\log(p(s) / (1-p(s))) = \alpha + \beta s \quad (18)$$

$$\log(p(c) / (1-p(c))) = \alpha + \phi c \quad (19)$$

Step 2

For each model compute the lack of fit statistic G^2 (with corresponding d.f.)

Denote G^2 by G_i^2 for model (16+i), $i=1, 2, 3$

Step 3

- 3a. If G_2^2 gives a good fit, then soak time is important independent variable
- 3b. If G_3^2 gives a good fit, then cycle time is important independent variable

Step 4

- 4a. To test the hypothesis of 'no cycle time effect'
compute $G_2^2 - G_1^2$ [increase in G^2 in using model (18) instead of (17)]. This difference is asymptotically distributed chi-squared with 1 d.f.
- 4b. To test for 'no soak time effect' compute $G_3^2 - G_1^2$, again this difference is asymptotically distributed chi-squared with 1 d.f.

Step 5

If both hypotheses in step 4a and 4b are rejected and G_1^2 indicates a good fit, then we conclude that both independent variables are contributing to failure.

For the remainder of this section, let the variable T denote the Soak time and the variable Z denote the cycle time. Various models developed in this report can be used to estimate the reliability of RCS thruster valves. In particular, if T is weibull and z has a gamma distribution, then the hazard rate is given by equation (4).

As mentioned before, with the advent of extended EVA's and docking, a growing interest in the field performance of RCS thruster valves has developed. The reliability and safety requirements for the space shuttle program, have emphasized the need for adequate statistical methods for obtaining reliable safety guarantees. To achieve this objective, we need reliable sample data from various RCS thruster valves systems. Available field failure data on these valves is from a number of individual systems, each characterized by a serial number. The system may be put into, or taken out of, operation at different times. For each thruster valve, its (censored) life history which contains the following information is available:

- The time when the valve was put into operation.
- The location of the valve on the orbiter.
- New or flushed valve
- The time when the valve failed.
- The time periods when the system was temporarily put out of operation (down periods).

The location information is necessary if the objective is to find a location (or locations) on the orbiter where valves are more likely to fail.

The Maximum Likelihood procedure is a powerful method of estimating parameters in

statistics. However, due to censoring, the likelihood function in this case will include:

- a. a set of lifetime observations

$$T_1, T_2, T_3, \dots, T_{N1}$$

for those valves which have been replaced after failing; and

- b. a set of right censored observations

$$T_1^*, T_2^*, T_3^*, \dots, T_{N2}^*$$

for the valves that survived the flight time. Then the estimation is equivalent to fitting the parameters to the mixture distribution, so that the estimate $\theta = (\beta, \eta, \lambda, \gamma)$ is obtained as the value of θ that maximizes the log - likelihood expression

$$L(\theta) = \sum_{j=1}^{N1} \log f(T_j) + \sum_{i=1}^{N2} \log [1 - F(T_i^*)].$$

In this case $L(\theta)$ is quite complicated because of two types of observations (T_i 's and T_i^* 's), and excessive number of parameters in the model. This is an interesting problem. I plan to continue working on this problem and try to complete the problem during summer 96.

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